Galactic and Extragalactic Propagation of Cosmic Rays

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(Received 6 April 1964)

A model for the origins of cosmic radiation is examined in which nonsolar primaries below 10^{17} eV are assumed to come from supernovae within the spiral arms of our galaxy, and those at higher energies are attributed to extragalactic sources. Supernova-produced particles are taken as diffusing first in the spiral arm region, then leaking into the galactic halo where they travel with a larger diffusion mean free path, and eventually diffusing into extragalactic space. These particles would be observed with the characteristic energy and mass spectra with which they are injected into the spiral arms, except in the energy range 10^{15}–10^{17} eV where they begin to see longer effective mean free paths and to escape more easily from the spiral arms. The energy spectrum is consequently steepened in this range and the abundance of primaries shifted towards heavier nuclei. Above approximately 10^{17} eV the flux from supernovae falls below that of another cosmic-ray population originating in extragalactic sources and taken as diffusing throughout the local supercluster of galaxies. The lifetimes of these primaries are sufficiently long that most of the higher energy particles will have photodecayed into protons. Parameters in the computations are chosen to fit cosmic-ray observations, to minimize the total cosmic-ray energy required, and to conform reasonably with current astronomical speculation. They result in a flux in the galactic halo almost one order of magnitude less intense than in the spiral arms, and that in the supercluster almost another two orders of magnitude lower. The galactic sources are required to furnish an average of over 10^{48} ergs per year in cosmic-ray energy.

I. INTRODUCTION

THEORIES of the origins of cosmic rays have suffered from a shortage and ambiguity of necessary data. Recent experimental results are beginning to allow a detailed study of the differences between alternative approaches. This paper is intended to compare these results with predictions of an eclectic diffusion model for the origins of cosmic rays.

Many physicists have attempted to ascribe the origin of nonsolar cosmic rays to a single type of source. For example, Ginzburg and Syrovatsky[1,2] suggest that they may all come from supernovae in our galaxy; Sciama[3] assumes that we observe particles from supernovae in our local cluster of Galaxies; Burbidge[4] discusses the possibility that they originate in strong radio sources within our supercluster of galaxies; and Burbidge and Hoyle[5] speculate that all of the universe may be full of cosmic rays accelerated by a Fermi process on an extragalactic scale. However, Morrison[6] and others propose eclectic models in which different sources contribute particles at different characteristic energy ranges.

One major objection to an eclectic theory lies in the apparent simplicity and unity of the data on cosmic rays, especially in the energy spectrum. For a long time this has seemed consistent with a particle intensity $N(>E)=KE^{-n}$ where n is constant at approximately 1.5 over an energy range of 10^{10} for particles ranging from protons to nuclei as heavy as iron.

All quantitative theories have predicted characteristic deviations from this simple spectrum. Recent observations indicate that such deviations are becoming apparent and permit the alternative models to be examined more closely. As is shown below, they also allow an eclectic model to seem more viable than in the recent past.

II. RECENT EXPERIMENTAL RESULTS

1. Energy Spectrum

Linsley[7] has confirmed earlier indications that the cosmic-ray energy spectrum has a kink at approximately 10^{15} eV. He finds that when one plots the logarithm of the total particle energy, the slope steepens appreciably (rising from 1.5 to approximately 2.4) in the range 10^{15}–10^{17} eV. However, the curve seems to flatten again above 10^{17} eV, and may reach 1.6 or lower at 10^{19} eV.

2. Composition of Primaries

The vast majority of cosmic-ray primaries have long been known to be protons. In studies of primaries coming from the sun, Biswas *et al.*[8] have gathered evidence that the relative abundances of atomic nuclei closely resemble the solar atmospheric abundances of

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* This research was supported in part by the National Aeronautics and Space Administration.

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[1] V. L. Ginzburg, in Progr. Elem. Particle Cosmic Ray Phys. 4, p. 339 (1957).

[2] V. L. Ginzburg and S. I. Syrovatsky, Progr. Theoret. Phys. (Kyoto) **20**, Suppl. 1 (1962). This paper also considers the possibility that the highest energy particles may be extragalactic.

[3] D. W. Sciama, Monthly Notices, Roy. Astron. Soc. **123**, 317 (1962).

[4] G. R. Burbidge, Progr. Theoret. Phys. (Koyto) **27**, 999 (1962).

[5] G. R. Burbidge and F. Hoyle (to be published).

[6] P. Morrison, in *Handbuch der Physik*, edited by S. Flügge (Springer-Verlag, Berlin, 1961), Vol. 46.

[7] J. R. Linsley, Report at the International Conference on Cosmic Rays, Jaipur, 1963 (to be published).

[8] S. Biswas, C. E. Fichtel, D. E. Guss, and C. J. Waddington, J. Geophys. Res. **68**, 3109 (1963).

the elements. Nonsolar primaries do not show this same pattern. They have a higher proportion of heavy nuclei and a very much higher proportion of Li, Be, and B.[9] While protons still predominate, the relatively higher incidence of heavies is reminiscent of the abundances in supernovae,[1] and the proportion of Li, Be, and B encourages estimates that these primaries have traversed approximately 2.5 g of matter cm^{-2} before reaching the earth. Since the overwhelming majority of cosmic-ray primaries have energies below 10^{11} eV, these data are best established at such relatively low energies. However, the pattern mentioned above seems to be followed up to energies of almost 10^{15} eV. (It is worth stressing, however, that above about 10^{14} eV the quoted primary spectra do not refer to energy per nucleon, but rather to energy per particle. Thus, heavy nuclei make a significant contribution to the flux of particles with a given total primary energy.)

Recent observations indicate a change at higher energies. Linsley and Scarsi[10] have found strong evidence that the primary particles at energies above 10^{17} eV are almost all protons. The composition of primaries in the crucial range between 10^{15}–10^{17} eV is much less certain. Zatsepin *et al.*[11] report that the composition seems to be essentially the same as at lower energies, but McCusker[12] observes a marked shift to heavier nuclei in that range.

III. AN ECLECTIC MODEL

In the model studied, most cosmic rays observed at the earth are assumed to originate in supernovae within the galactic spiral arms. They diffuse within that space until they leak into the galactic halo. The particles continue diffusing in the halo, but with a different mean free path, and eventually leak out into the region defined by the local supercluster of galaxies. Diffusion continues in that space with a still different mean free path, until finally the cosmic rays diffuse into general extragalactic space. Particles from the many similar galaxies in the supercluster are considered to behave essentially the same as those from our galaxy. However, approximately 10 very strong radio sources are now active in the supercluster.[4] These are considered as likely suppliers of the very high-energy cosmic rays which cannot easily be attributed to supernova sources.

The computations below assume that the cosmic-ray intensity in general extragalactic space is significantly lower than in our galaxy or even in the supercluster. Thus, one can apply diffusion equations with the condition that the density falls quite low at the boundary of the supercluster. This assumption is by no means necessary. One can suggest, as Hoyle and Burbidge do in part of their paper,[5] that the universe is full of cosmic rays at a density comparable to that observed at the earth. In a sense this paper is less bold. It attempts to fit current cosmic-ray observations with a model which minimizes the total energy in the universe given over to cosmic rays.

Exact solutions of diffusion equations depend sensitively upon the choice of boundary conditions.[13] However, order of magnitude calculations can be made easily on the basis of general considerations.

A particle diffusing with scattering mean free path λ and velocity v in a region whose smallest dimension is L will leak out of that region in time $t \cong L^2/\lambda v$. If these particles are generated fairly uniformly at power P within that region, and are not significantly accelerated after injection, the total energy stored within the region (of volume V) will be approximately $P(L^2/\lambda v)$ and the mean energy density will be $\bar{\varphi} = Pt/V \cong (P/V)(L^2/\lambda v)$.

In sample computations one can take the spiral arm as a curved cylinder with a mean radius of approximately 10^3 light years and a length of about 10^5 light years. The halo is considered almost spherical with a radius of approximately 5×10^4 light years. The supercluster is taken to have a mean radius of approximately 3×10^7 light years and to contain 10^4 galaxies, perhaps 10 of which are very strong radio sources.

The relevant parameters used are listed in Table I.

TABLE I. Parameters used in computations.

Region	L (light years)	λ (light years)	$\bar{H}$ (G)	$\bar{\rho}$ (g cm^{-3})
1. Spiral arm	10^3	10	5×10^{-6}	2×10^{-24}
2. Halo	5×10^4	10^2	2×10^{-6}	10^{-26}
3. Supercluster	3×10^7	$\leq10^5$	4×10^{-7}	10^{-29}

They are chosen to fit cosmic-ray observations, to minimize the total cosmic-ray energy required, and to conform reasonably with current astronomical speculation. The sensitivity of the computations to the precise choice of parameters is discussed below.

In region 1 the diffusion lifetime is $L^2/\lambda_1 v_1 = 10^5$ years. In this time, cosmic rays will have passed through $\bar{\rho} c t_1 = 0.2$ g cm^{-2}. This is not enough to change appreciably the mass spectrum of the particles. They also do not have enough time to be accelerated significantly in the arms by a Fermi process. Thus, this choice of parameters requires that they be injected into the spiral arms with their characteristic abundances and energies. They must therefore pass through almost 2.5 g cm^{-2} within the sources themselves.

If the observed energy density near the earth is taken as characteristic of the spiral arms (this is reasonable

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[9] F. W. O'Dell, M. M. Shapiro, and B. Stiller, J. Phys. Soc. Japan, Suppl. A-III 23 (1962).

[10] J. Linsley and L. Scarsi, Phys. Rev. Letters **9**, 123 (1960).

[11] G. T. Zatsepin *et al.*, Report at the International Conference on Cosmic Rays, Jaipur, 1963 (to be published).

[12] C. B. A. McCusker *et al.*, Report at the International Conference on Cosmic Rays, Jaipur, 1963 (to be published).

[13] See, for example, P. Morrison, in *Handbuch der Physik*, edited by S. Flügge (Springer-Verlag, Berlin, 1961), Vol. 46; or H. Laster, Phys. Rev. **107**, 1112 (1957).

unless the earth is within a few mean free paths of a boundary or a source), then $\bar{\varphi}_1$ is of the order of 10^{-12} erg cm^{-3}.

$$P_1 = \bar{\varphi}_1(V_1/t_1) \cong 3\times 10^{48} \text{ erg year}^{-1}.$$

If supernovae are assumed to occur approximately once per hundred years, each must inject an average of approximately 3×10^{50} erg of cosmic-ray energy into galactic space. This number is somewhat larger than Ginzburg's,[2] but compatible with it. It is completely consistent with some recent theories of supernovae.[14]

Particles from the spiral arms then diffuse out to the halo. It is assumed that there are no very significant sources of new cosmic rays in that region. The diffusion lifetime there is $(L_2^2/\lambda_2 v_2) = 2.5\times 10^7$ years. In this time the cosmic rays will pass through another 0.25 g cm^{-2} of matter, which does not affect significantly their mass spectrum. The energy spectrum similarly will be unchanged.

However, the energy density in the halo is reduced by almost an order of magnitude

$$\bar{\varphi}_2 \cong P_2(t_2/V_2) = P_1(t_2/V_2) \cong 1.5\times 10^{-13} \text{ erg cm}^{-3}$$
$$\bar{\varphi}_2 \cong 0.15\bar{\varphi}.$$

(The actual cosmic-ray intensity is somewhat greater near the spiral arms and lower near the outer boundary of the galactic sphere.)

The particles eventually diffuse into region 3. If the diffusion mean free path is no larger than 10^5 light years (this is a reasonable upper estimate for cosmic rays at very high energies, as is discussed below), they remain in region 3 for times comparable to its age.

$$t_3 \cong (L_3^2/\lambda_3 v) \geqslant 10^{10} \text{ years}.$$

In such times the cosmic rays will pass through perhaps another 0.1 g/cm^2. The mass spectrum of lower energy particles remains unchanged, but most cosmic rays of energy greater than 10^{17} eV will have interacted with photons Doppler shifted to energies above 10 MeV, as suggested by Morrison,[6] Ginzburg,[2] and others. As a result of photodecay, most primaries above 10^{17} eV should be protons. (The computations above have assumed a mean density of 10^{-29} g cm^{-3} in the supercluster. If the actual gas density were two orders of magnitude higher, nuclear collisions would also play a significant role. In that case, the decay of heavies into protons in region 3 would also be important at energies below 10^{17} eV.)

If 10^4 galaxies in the supercluster make contributions similar to that of our galaxy, the resultant cosmic-ray energy density in region 3 is approximately $2\frac{1}{2}$ orders of magnitude lower than in the spiral arms

$$\bar{\varphi}_3 \cong P_3(t_3/V_3) \cong 10^4 P_1(t_3/V_3) \cong 3\times 10^{-15} \text{ erg cm}^{-3}$$
$$\bar{\varphi}_3 \cong 3\times 10^{-3}\bar{\varphi}.$$

As can be seen above, the cosmic-ray intensity from supernova sources falls sharply as one moves from the region 1 to region 2 to region 3. Although the diffusion model allows particles from the larger volume to diffuse into a smaller region, their lower intensity reduces the significance of their contribution. Thus, for example, one can expect low-energy cosmic rays from other galaxies to make, at most, a minor contribution to the flux observed at the earth.

However, the calculations above are not appropriate at high energies. The estimated diffusion mean free paths are not valid over 10^{15} eV, as is shown below. In addition, it is difficult to imagine mechanisms for generating in supernovae cosmic rays at energies up to 10^{20} eV, and it is essentially impossible to contain such particles within the spiral arms.

For these reasons, we assume that other sources of high-energy cosmic rays must exist. As Morrison[6] and Burbidge and Hoyle[4,5] point out, strong extragalactic radio sources seem capable of supplying cosmic rays. They generate sufficient energy to make significant contributions to the extragalactic density, and they are large enough to be considered as accelerators of very high-energy particles. Hoyle and Burbidge[5] calculate that the strong radio sources within a volume of 10^{81} cm^3 [10^{27} (light years)^3] can supply sufficient cosmic rays to fill that volume with an energy density of 10^{-14}–10^{-12} erg cm^{-3}. The lower figure is consistent with estimates of intergalactic cosmic-ray fluxes based on Felten and Morrison's[15] study of the production of γ rays outside the galaxy.

It consequently seems plausible that strong radio sources would fill region 3, and perhaps even all of extragalactic space, with cosmic rays at an energy density of 10^{-14} erg cm^{-3}. If the integral energy spectrum has the same slope as that of galactic cosmic rays, these particles should not be noticeable at the earth unless the galactic spectrum steepens its slope before the extragalactic one. If the region 3 spectrum is very much steeper, the extragalactic particles may never be noticeable except indirectly. If the spectrum is flatter, the two curves may intersect without requiring a change of slope, and most cosmic rays above the energy at intersection would be from extragalactic sources. This last possibility also allows these sources to make particularly significant contributions to the high-energy spectrum of cosmic rays without requiring enormous total energies in the form of cosmic rays.[16] It is a very attractive possibility, since there is tentative evidence

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[14] F. Hoyle (private communication).

[15] J. E. Felten and P. Morrison, Phys. Rev. Letters **10**, 453 (1963).

[16] One can imagine, for example, two fluxes of cosmic rays with integral spectra $J_1 = K_1 E^{-1.5}$ and $J_2 = K_2 E^{-1.2}$ over a range $2.5\times 10^9 \text{ eV} \leq E < \infty$. The ratio of total energy density from flux 1 to that from flux 2 is $(K_1/K_2)(0.2/0.5)(E_0^{0.5}/E_0^{0.2})$, where $E_0 = 2.5\times 10^9$ eV. If this is 10^2, the two spectra will intersect at 2×10^{17} eV. If it is 10^3, they will intersect at 4×10^{20} eV. Any dip in the first spectrum could, of course, produce an intersection at an even lower energy.

that some strong radio sources such as 3C 273 have rather flat radio spectra,[17] and therefore might have flatter cosmic-ray spectra. However, the connection between radio spectrum and cosmic-ray spectrum is still uncertain for strong radio sources. Other studies[18] indicate that a slope of 1.5 in the energy spectrum may be natural in major sources of cosmic rays. As a result, it seems reasonable to assume similar energy spectra from different sources and to explain a possible shift from a galactic to an extragalactic flux as being due to a change in the energy spectrum of the former.

Such a change seems very likely. Protons above 10^{15} eV cannot easily be confined within the spiral arms. Byakov[19] has pointed out that their effective diffusion mean free path will increase with energy beyond the 10 light years chosen. As it does, the storage life time goes down and the proton flux falls more rapidly than in the injection energy spectrum. Eventually it should fall to the level of the halo flux and then below that as the effective mean free path in the galaxy increases beyond the 100 light years assumed in the halo.

However, the Larmor radius for nuclei of the same energy varies as $1/Z$. Thus, heavy primaries will first experience a change of mean free path at higher energies than protons.

Approximately 50% of the cosmic-ray primaries at total energy 10^{14} eV and above are alpha particles and heavier nuclei. One can expect that at about 10^{15} eV the energy spectrum of protons will begin to fall rapidly as the mean free path increases, and that this steepening of the spectrum will occur at progressively higher energies for the heavier primaries. Thus, the primary energy spectrum will develop a kink at about 10^{15} eV, and also will display a marked shift to heavier nuclei as the change of mean free path is experienced by heavy primaries at increasingly higher energies.

At some high energy the intensity of galactic cosmic rays may fall below that in extragalactic space. As was shown above, this latter flux can be attributed to sources other than supernovae at very high energies. The maximum diffusion mean free path chosen for region 3 would seem appropriate for particles at energies 10^{17}–10^{20} eV. These particles will be confined to the supercluster for approximately 10^{10} years, and consequently will have decayed mainly into protons.

In summation, this model attributes nonsolar cosmic rays incident on the earth to two different sets of sources. Below approximately 10^{17} eV we primarily see particles originating in supernovae in our galaxy and diffusing within the spiral arms and the galactic sphere. These particles retain the characteristic energy spectrum and mass spectrum with which they are injected into the spiral arms, except in the energy range 10^{15}–10^{17} eV, where a change in effective mean free path steepens the spectrum and shifts the relative abundances towards heavy elements. At about 10^{17} eV the flux from supernovae falls below that from extragalactic sources. Thus, primaries between 10^{17}–10^{20} eV can be expected to have an energy spectrum characteristic of the extragalactic sources and to consist mainly of protons as a result of photodecay of heavies diffusing for lifetimes of the order of 10^{10} years.

The resultant cosmic-ray intensities vary in different regions of space. In Table II they are shown to be consistent with the magnetic field energies as computed from the choice of parameters above.

TABLE II. Cosmic-ray and magnetic-field energy densities.

Region	Cosmic-ray energy density (eV cm^{-3})	($H^2/8\pi$) (eV cm^{-3})
1. Spiral arm	10^{-12}	10^{12}
2. Halo	1.5×10^{-13}	1.6×10^{-13}
3. Supercluster	3×10^{-15}–10^{-14}	6×10^{-15}

IV. COMPARISON WITH OBSERVATIONS

1. Energy Spectrum

Linsley[7] describes a primary particle energy spectrum which has a characteristic slope up to approximately 10^{15} eV, steepens in the range 10^{15}–10^{17} eV, and seems to flatten again at about 10^{17} eV. This is completely consistent with the above model. The slope below 10^{15} eV is attributed to the supernova sources; the kink at 10^{15} eV occurs as first protons, and later heavier primaries, experience an effective increase in diffusion mean free path; the second kink at about 10^{17} eV occurs as the flux from supernovae falls below that from extraglactic sources. The specific slope of approximately -1.5 below 10^{15} eV has been explained in terms of equipartition among different forms of energy in supernova energies. Such arguments can be extended to the extragalactic sources to predict a slope of -1.5 at the highest energies. However, this slope may also steepen near or above 10^{20} eV as particles begin to escape more easily from region 3, or the sources are unable to accelerate particles to higher energies. Alternatively, one might expect a still gentler slope than -1.5 at the highest energies if flat radio spectra can be interpreted as implying flat cosmic-ray spectra from some sources. If such a flat slope is clearly established, it will provide very strong evidence that the highest energy particles have different origins from those at lower energies. The slope in the range 10^{15}–10^{17} eV certainly should be steeper than -1.5. Byakov[20] computes that it should not be steeper than -2.5, which is consistent with tentative observations. However, 10^{15}–10^{17} eV is clearly a transition range; at the lower energies one begins to observe the

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[17] R. G. Conway, K. I. Kellerman, and R. F. Long, Monthly Notices Roy. Astron. Soc. **125**, 261 (1963).

[18] S. I. Syrovat-Skii, Zh. Eksperim. i Teor. Fiz. **40**, 1788 (1961) [English transl.: Soviet Phys.—JETP **13**, 1257 (1961)].

[19] V. H. Byakov, Astron. Zh. **40**, 625 (1963) [English transl.: Soviet Astron.—AJ **1**, 480 (1964)].

effects of a loss of supernova particles, and at the higher end a second population of cosmic rays beings to be observed. It consequently is still difficult to speak meaningfully about the exact slope in that range.

2. Composition of Primaries

The relative abundances of elements among cosmic-ray primaries below 10^{15} eV is consistent with a theory of supernova origins.[1,2] The abundances of Li, Be, and B indicate that these cosmic rays have traversed approximately 2.5 g/cm^2 of matter. Since the computations above do not store the particles sufficiently long in the galaxy for this to occur, the region in which limited spallation occurs would seem to be in the supernova sources themselves.

This diffusion model closely associates a steepening of the energy spectrum with a shift in the mass spectrum of primaries towards heavies. This is consistent with McCusker's results[12] in the range 10^{15}–10^{17} eV and inconsistent with Zatsepin's.[11] (Confirmation of the latter would seriously weaken the model.)

Above 10^{17} eV the model predicts a shift to a second population of cosmic rays. These particles are, on the average, several orders of magnitude older than the supernova-produced cosmic rays. They have had time to experience photodecay or perhaps nuclear collisions, and should consist primarily of protons. This is in agreement with Linsley and Scarsi's results.[10]

V. DISCUSSION

The calculations above are intended to illustrate the continued viability of an eclectic diffusion theory approach to the origins of cosmic rays. Some details are basic to this point of view. However, others are less basic and are included primarily to show how a detailed picture can be created.

1. Basic Approach

This model assumes that most observed cosmic rays originate in supernovae within our galaxy. They are injected with characteristic energy and mass spectra into a region where their diffusion provides mixing and storage. Mixing is necessary to produce isotropy, storage to reduce the cosmic-ray energy production demanded of the sources.

At some energy a break can be expected in the smooth energy spectrum as particles of high rigidity see longer effective mean free paths, and also escape more easily from the borders of the region. Thus, a shift in primary abundances towards heavies should coincide with the steepening of the energy spectrum. At a still higher energy the flux from supernovae falls below that in a second population of cosmic rays. These are assumed to come from another set of sources, and to have significantly longer lifetimes; consequently, they have a different mass spectrum.

2. Choice of Parameters

As described immediately above, this model does not require three distinct regions of diffusion with clear-cut boundaries. Indeed, common sense argues against such an oversimplification, since one can expect a gradual transition in physical characteristics as a boundary is crossed.

However, it seems reasonable to consider such distinct regions for the purposes of computation. The results depend most sensitively on the choice of parameters in the spiral arms. The diffusion mean free path is taken to be 10 light years so as to explain the kink in the energy spectrum at 10^{15} eV and to minimize the cosmic-ray energy required of supernovae. A smaller mean free path probably contradicts current tentative astronomical measurements of spacing and size of interstellar gas clouds. It also would produce a kink at significantly lower energies, unless the spiral arm magnetic field is taken to be much higher than most estimates allow. A larger mean free path would reduce the storage lifetime of cosmic rays, and consequently increase further the already high estimates of cosmic-ray production is supernovae. (This could be avoided by enlarging the dimensions of region 1, but then the location of the kink at 10^{15} eV would require a significantly lower magnetic field.)

If the parameters chosen for region 1 are appropriate, the characteristics of regions 2 and 3 seem to be much less significant for matching current data. For example, the diffusion mean free path in the galactic halo could be much longer. This would not alter noticeably the fit with cosmic-ray data; however, it would predict significantly lower cosmic-ray densities in the region and consequently suggest that radio frequency radiation from the halo should be almost unobservable. (If, as some speculate, the galactic halo is transient, region 2 could even be eliminated from these computations without changing seriously the basic conclusions of the model. At the moment, however, it seems premature to examine this possibility in detail.) Similarly, the existence of the supercluster of galaxies as a significant diffusion region may be unnecessary if one allows a second population of cosmic rays to fill the whole universe with high-energy particles at a density of $10^{-15}-10^{-14}$ eV cm^{-3}. Alternatively, one might consider region 3 as a much smaller local cluster of galaxies, as in Sciama's work.[3] However, such a change would require the magnetic field strength in the region to be increased and the mean free path decreased so that high-energy particles can be stored for times of the order of 10^{10} years. Sciama and others increase storage by allowing reflection of cosmic rays approaching the boundary of a region. This is consistent with some astrophysical descriptions of galactic or cluster magnetic fields, and it fits well with Sciama's assumption of invariance of each particle's magnetic moment. However, it seems somewhat arbitrary in a diffusion model, which depends upon isotropic scattering of most particles off magnetic inhomogeneities. As a result, no such reflection is assumed in this paper.

3. Conclusion

It is not yet possible to choose confidently between various theories of cosmic-ray origins. Current cosmic ray data can be reconciled with many. Even the prediction of a break in the energy and mass spectra between 10^{15}–10^{17} eV can be obtained from other theories. It comes from the assumption of rigidity dependence in cosmic-ray propagation. Any model which allows primaries of rigidity 10^{15} V to begin escaping prematurely will predict a significant steepening of the energy spectrum and a shift in abundances to the heavies. (This process could occur in the sources themselves or in the diffusion region.) If this model allows a second less intense population of particles extending to 10^{20} eV, current data can be fairly well fitted.

However, many other approaches can be used, the eclectic diffusion model seems to be worth exploring further. In recent years persuasive arguments against an eclective model have questioned the smooth fit of different fluxes into a single straight energy spectrum. It is now becoming increasingly clear that fine structure in the energy and mass spectra can be found. These weaken the arguments against an eclectic approach and suggest the advantages of carefully reexamining it.

ACKNOWLEDGMENT

The author is grateful to the Department of Applied Mathematics and Theoretical Physics, University of Cambridge, Cambridge, England, for its hospitality while this work was done. He thanks scientists at Imperial College, London and Cambridge—particularly Dr. H. Allan, Professor F. Hoyle, and Dr. D. Sciama—for stimulating conversations.

PHYSICAL REVIEW VOLUME 135, NUMBER 5B 7 SEPTEMBER 1964

Birefringence of the Vacuum

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(Received 27 April 1964)

It is known that Maxwell's equations become nonlinear if the effect of virtual electron-positron pair creation is included. The vacuum, thus behaving like a polarizable continuum, is shown to exhibit the phenomenon of birefringence.

THE fact that in quantum electrodynamics two photons can interact through the creation of virtual electron-positron pairs leads to additional terms in the Lagrangian density which are biquadratic in field intensities. It is shown in this note that this causes birefringence of the vacuum. An estimate of the Kerr constant of the vacuum gives a value of $(7/90\pi) \times (e^2/\hbar c)^2(1/mc^2)(\hbar/mc)^3\Lambda^{-1}$, where Λ is the wavelength of the light.

The interaction between two photons has been treated[1–5] in quantum electrodynamics by considering the production of virtual pairs in the vacuum. It is now well known that this phenomenon of scattering of one photon by another (Fig. 1) leads to a nonlinear interaction between electromagnetic fields in vacuum. The S-matrix element $(k_3,k_4|S|k_1,k_2)$ for the scattering of two photons of 4-momenta k_1 and k_2 into k_3 and k_4 can be written in the following manner:

$$(k_3,k_4|S|k_1,k_2)=\left(k_3,k_4\middle|-i\int \mathfrak{L}_{I\mathrm{eff}}d^4x\middle|k_1,k_2\right), \quad (1)$$

where the effective Lagrangian density $\mathfrak{L}_{I\mathrm{eff}}$ must satisfy the requirements of relativistic invariance and gauge invariance and hence must involve the invariants $(\frac{1}{2}F_{\mu\nu}F^{\mu\nu})^2=(\mathbf{B}^2-\mathbf{E})^2$ and $(\frac{1}{8}\epsilon_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma})^2=(\mathbf{B}\cdot\mathbf{E})^2$ of the electromagnetic field, where $F_{\mu\nu}=\partial_\nu A_\mu-\partial_\mu A_\nu$ and the indices μ and ν take the values 0, 1, 2, 3. According to Schwinger,[5] and Karplus and Neuman,[4] the result for Fig. 1 is given by

$$\mathfrak{L}_{I\mathrm{eff}}=\frac{2\alpha^2}{45(4\pi)^2m^4}[(\mathbf{B}^2-\mathbf{E}^2)^2+7(\mathbf{B}\cdot\mathbf{E})^2], \quad (2)$$

where the fine structure constant $\alpha=e^2$ (in naturalized Gaussian units, $\hbar=c=1$) and m is the mass of the electron. The complete Lagrangian for the electromagnetic field, including the effect of virtual pair

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[1] O. Halpern, Phys. Rev. 44, 855 (1933).
[2] H. Euler, Ann. Physik 26, 398 (1936).
[3] W. Heisenberg and H. Euler, Z. Physik 98, 714 (1936).
[4] R. Karplus and M. Neuman, Phys. Rev. 80, 380 (1950); 83, 776 (1951).
[5] J. Schwinger, Phys. Rev. 82, 664 (1951).